

**UNIT EC425, LEVEL 2**

**PRODUCTION**

**MICROECONOMICS 2:  
ECONOMICS AND ORGANISATIONS**

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## **Production**

### Introduction

The aims of this lecture are

- to demonstrate that the concept of the production function is applicable to a wide set of circumstances, not just the firm.
- consider the implications of the underlying assumptions associated with production functions.

To do this I shall

- describe production activities and processes
- examine the properties of production functions
- investigate the properties of technology by looking at its returns to scale and elasticity of substitution.
- present evidence to show the wide application of production function techniques.

### Organisations

“Even in the most libertarian of free-market economies, individuals voluntarily surrender their freedom of action in order to form and join organisations.”

Bruce Lyons in Hargreaves Heap *et al* (1993) *The Theory of Choice. A critical guide*, Blackwell, London. p155.

Organisations can take a variety of forms. The State is an embracing organisation for all individuals but there are a number of other organisational types that meet the every day needs of individuals: trade unions, clubs, charities, universities, hospitals, firms, etc.

### The General Production Function

The simplest way of viewing the organisation is as a ‘black box’



They create goods or services (output) using scarce resources (inputs).

We can use mathematical notation to describe the process. So,

$$X = f(L, K, v, \gamma)$$

Where

X = Output per period. (A flow)

L = Labour

K = Capital

where K & L are both flows over the long run

$v$  = returns to scale (refers to the long run)

$\gamma$  = efficiency factor ((refers to the entrepreneurial/ organisational aspect of the firm)

In organisations that produce tangible products (goods) there is little debate about what constitutes inputs and output. This is also true to a large extent in private sector service organisations. However, in public service organisations (e.g., local authorities or hospitals) there is debate in recent years about what constitutes output, some prefer the term outcomes. Similarly, there has been much discussion in public services about the impact of inputs on returns to scale and efficient practices.

As this lecture unfolds, it should become clear that the concepts involved in production functions have wide applicability, including our understanding of economic growth and development and the reasons why trade takes place.

### **Examples**

Most of the examples in boxes are taken from Mulligan J (1990) Managerial Economics.

***We'll have a nine-pounder please!***

Rosenweig MR & Schultz TP (1983) Estimating a household production function. Heterogeneity, the demand for health inputs and their effects on

birth weight, *J. of Pol. Economy*, 91, no. 5, pps. 723- 46.

Using data on 9,000 live births to married couples between 1967 and 1969 the authors hypothesised that an expectant mother might be able to choose inputs that could influence the weight and future health of her baby. Specifically,

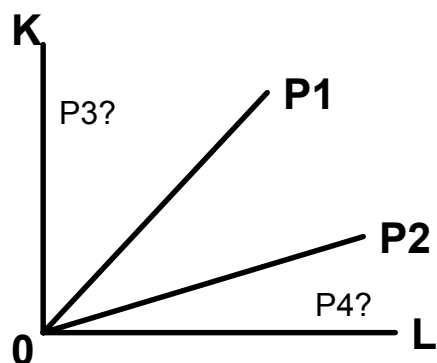
Birth weight = f (the no. of months before the first pre-natal visit, no. of cigarettes smoked per day, the order of the birth among total no. of births, and mother's age)

They found a significant relationship between the inputs specified and output. e.g.

- the typical smoker smoked 14 cigarettes per day during pregnancy and produced a baby who was 7% lighter than the average baby of a non-smoker.
- the optimal age for a mother was 24 years. At 20, babies BW was 4.4 % lower. At 30, babies BW was 6.7% lower.
- Age had an additional impact. Mothers who had their fourth child at 30 had a heavier baby than a 20 year old mum giving birth to her fourth child!
- Black babies had lower BWs than white babies due, it seems to shorter gestation period. The authors found that gestation period lengthened with pre-natal care and so suggested a policy promoting earlier pre-natal care for black mothers.

## Production Activities and Processes

The entrepreneur may well discover a number of ways of producing a particular product or service. That is they use capital and labour in various combinations to produce a given level of output.



A technology consists of all the processes available to produce a certain product. And a set of technologically feasible labour-capital combinations is referred to as a production set.

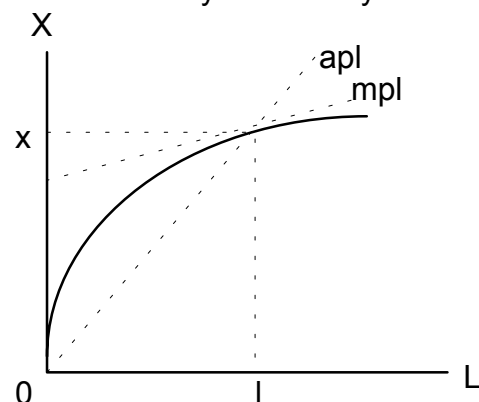
However, the entrepreneur may be constrained in the short run because a particular factor is fixed.

## The Short Run Production Function

In the short run the Law of Variable Proportions or Diminishing Returns (productivity) prevail. This states that if one of the

factors of production (usually K) is fixed, the MP of the variable factor (L) will diminish after a certain range of production (though there is the rare case where scale effects offset diminishing returns).

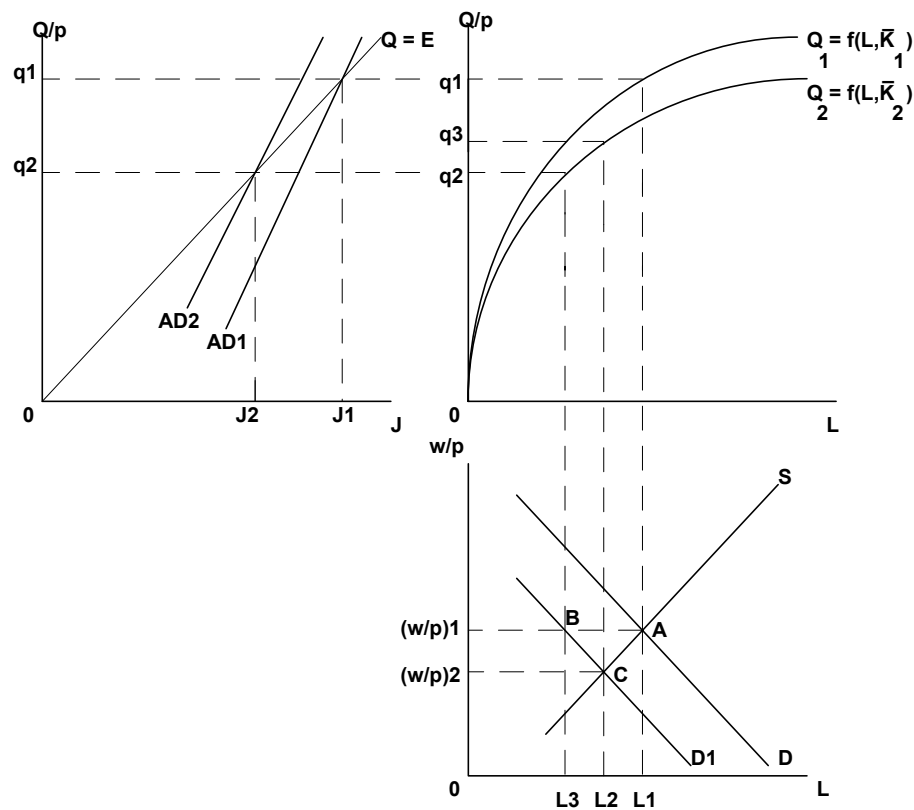
Note that with short run, two input, production functions factor substitutability is severely constrained.



We can see from the above total product curve that the average product of labour,  $X/L$ , is always above the marginal product of labour,  $\Delta X / \Delta L$ , though both are falling.

We can use the short run production function to illustrate issues such as plant closure within an area. The diagram below shows the inter-relationship between regional productivity, labour markets and the multiplier. So assuming that wages remain fixed, the Keynesian assumption, the regional/local multiplier effects of a plant closure would be greater than under neo-classical assumptions about labour

market flexibility. One also has to assume that the plant being closed is not unproductive in the first instance!



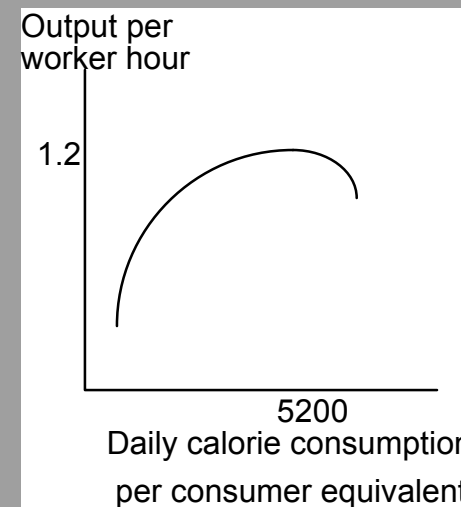
**A well fed worker is a productive worker**

Strauss J (1986) Does better nutrition raise farm productivity? *J. of Pol. Econ.* 94, no.2 pp. 297-320.

Economists are interested in the role that economic incentives play in an individual's choice of diet. The author analysed a data set of households in Sierra Leone, a small West African country, during the 1974-75 cropping year. The data also included information on the production and purchase of 196 foods eaten by farmers. Using conversion tables he calculated the caloric intake of each worker on the farm.

He hypothesised that

Agricultural output =  $f$  (labour hours, caloric intake of workers, fixed capital and land cultivated)



The above total product curve has a positive but declining slope until 5200

calories per day. That is, there is a diminishing marginal productivity of caloric consumption per day. The author calculated output elasticity showing that, at the average intake for the group with the lowest one-third consumption of calories (1,500 per day), the output elasticity was 0.49 (i.e. a 1% increase in caloric intake increased productivity by 0.49%). For the group with a caloric intake of 4,500 calories per day, the elasticity was 0.12.

### ***No more school holidays!***

Link C & Mulligan JG (1986), The merits of a longer school day, *Economics of Education Rev.*5, no. 4 pps 373-381.

There is a debate in most Western countries about how we should tackle maths and literacy standards. These authors recognised that there will eventually be diminishing returns from increasing the amount of maths and reading instruction, but was there any way of determining the important variables.

Analysing the maths and reading scores of 118,000 US elementary schoolchildren during the school year 1977-78 they hypothesised

Test score =  $f$  (Time spent studying per day, time spent studying per year, participation of parents, efforts and abilities of teachers)

Specifically, they found that

- the marginal product of increasing instruction by one hour a day was essentially zero.
- test scores improved if the school year was lengthened.

## **The Long Run Production Function**

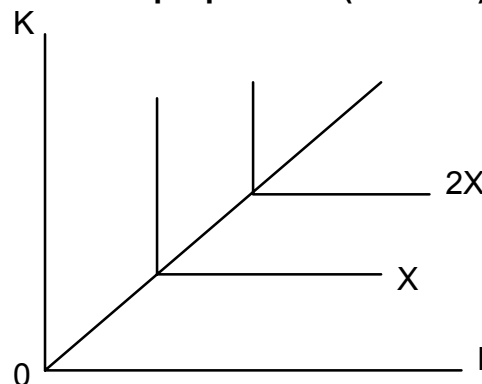
In the long run all factors of production are variable.

### **a) Substitutability of Factors**

To what extent can L be substituted for K, or vice versa, holding output constant?

There are 2 limiting cases.

- **Fixed proportions (Leontief) technology.**

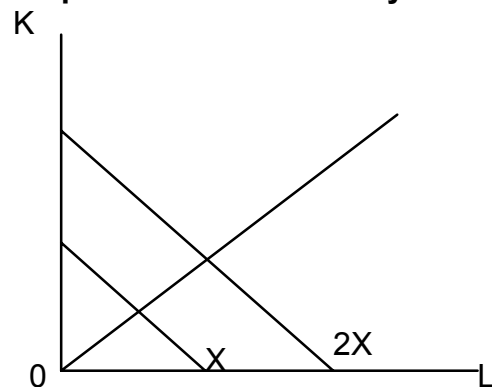


$$X = f(K, L) = \min(K, L)$$

The operator 'min' means that X is given by the smaller of the 2 values in parentheses. EG., a firm has only one production activity available to it and it can produce one unit of X with multiples of 3 K and 5 L (ie, in fixed proportions). However, if

it only has 25 units of L available to it will only be able to produce 5X, even if it has 21 units of K. In this case there is no substitutability between K and L.

• **perfect substitutability**



$$X = f(K, L) = aK + bL$$

Here it is possible to substitute perfectly, e.g. if  $X = K + 2L$  then the Marginal Rate of Technical Substitution of labour for capital = 2. That is, for X to remain constant the firm can give increase (decrease) labour by 1 unit if it decreases (increases) capital by 2 units.

• **The usual case**

The usual case, however, is where the isoquant is convex to the origin (see the Cobb- Douglas function below for one form of convexity). The slope of the isoquant shows the marginal

rate of technical substitution between K and L.

So the  $MRTS_{L,K}$  shows the amount of K that a firm can give up by increasing the amount of L by one unit and still produce the same output.

$$MRTS_{L,K} = -\frac{\partial K}{\partial L} = \frac{\partial X / \partial L}{\partial X / \partial K} = \frac{MP_L}{MP_K}$$

The slope of the curve is an unsatisfactory number by which to characterise the ease of substitutability because it is not independent of the units of measurement. To do this we need the **elasticity of substitution** concept.

Assuming that isoquants are convex, the  $MRTS_{L,K}$  will decline in algebraic terms, and increase in absolute value, as L is substituted for K along the isoquant. At the same time, the KL ratio will be diminishing. The elasticity of substitution is a pure number, defined by the proportionate change in factor proportions (K/L) that takes place around the isoquant, divided by the proportionate change in the  $MRTS_{L,K}$  that simultaneously occurs. Thus,

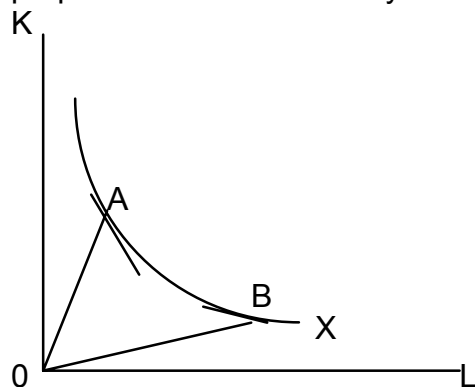
$$\sigma = \frac{\% \Delta in(K / L)}{\% \Delta in MRTS_{L,K}}$$

or

$$\sigma = \frac{\frac{\partial (K / L)}{K / L}}{\frac{\partial (\partial K / \partial L)}{\partial K / \partial L}}$$

Intuitively, the easier it is to substitute K for L, the less will be the proportionate change in the  $MRTS_{L,K}$  as the KL ratio falls, and the greater will be the absolute value of  $\sigma$ .

In the special case of perfect substitutability and fixed proportions  $\sigma$  will be infinity and zero respectively.



In moving from A to B on the X isoquant, both the KL ratio and the MRTS will change. The elasticity of substitution is defined to be the ratio of these proportional changes. It is a measure of how curved the isoquant is.

### **b) Returns to scale: Homogeneity and Homotheticity**

There are special types of functional relationships for which what is true of one section of one ray through the origin is true of all sections of all rays. One class of such functions is called homogeneous functions.

A production function is said to be homogenous if, when each input factor is multiplied by a positive real constant k, the constant can be completely factored out.

So if

$$X_0 = f(K, L)$$

$$X^* = f(kK, kL)$$

if k can be factored out then

$$X^* = k^v f(K, L)$$

or

$$X^* = k^v X_0$$

This is a homogeneous production function.

If  $v = 1$  then constant returns exist. The production function is homogeneous of degree 1.

If  $v > 1$  then increasing returns prevail

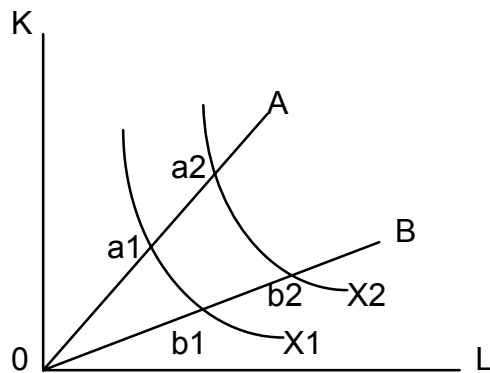
If  $v < 1$  then there are decreasing returns to the inputs.

Homogeneity in the two input case implies two characteristics.

- all isoquants that cut any one ray through the origin do so with the same slope. This is Homotheticity.

Homothetic isoquants that are equidistant from one another along any one ray are also equidistant along any other - there is a definite cardinally measurable change in the level of output as we move to a new isoquant.

- it displays the same returns to scale characteristics along any ray regardless of the input combination represented by that ray.



The above diagram can be used to illustrate the concepts of homogeneity and homotheticity.

A homothetic production function arises

- If the slope of X1 at a1 = slope of X2 at a2, and similar

reasoning is applied to to b1 and b2.

In addition, the production function is homogeneous

- if  $\frac{\partial a_2}{\partial a_1} = \frac{\partial b_2}{\partial b_1}$

The convenience of homogeneity is this. It provides a simple summary statistic that describes the way in which output changes with all inputs. This statistic is independent of both the level of output used and the KL ratio.

Homotheticity is a more general case because it is showing where it is technically possible to increase output whilst keeping the MRTS constant, i.e. it shows non-homogenous production function.

Homogeneity is a very strong assumption to make about the nature of technology. Indeed, traditional long run cost curves depicts variable returns to scale. Though there has been a lot of support for the constant returns case following the work of Cobb and Douglas back in the early part of the century.

### Cobb-Douglas Production Functions

From the functional form

$$X = aK^b L^c$$

we find the following properties:

- **The function is intrinsically linear** ( $\log X = \log a + b \log K + c \log L$ )

- **the parameters a, b, and c can be estimated by regression**

- **The MP of each factor is equal to the factor exponent times its average product e.g.**

$$\begin{aligned} MP_K &= \frac{\partial X}{\partial K} = b \cdot a \cdot K^{b-1} L^c \\ &= b(a \cdot K^b \cdot L^c) K^{-1} \\ &= b \frac{X}{K} \end{aligned}$$

- **The MRTS is equal to the ratio of exponents times the factor utilisation ratio, e.g.**

$$MRTS_{L,K} = \frac{\partial X / \partial L}{\partial X / \partial K} = \frac{c(X/L)}{b(X/K)} = \frac{c}{b} \cdot \frac{K}{L}$$

- **The output elasticity with respect to any input is equal to the exponent of that input. e.g.**

$$e_L = \frac{\partial X}{\partial L} \cdot \frac{L}{X}$$

as

$$\frac{\partial X}{\partial L} = MP_L = c \left( \frac{X}{L} \right)$$

so

$$e_L = c \left( \frac{X}{L} \right) \cdot \left( \frac{L}{X} \right)$$

$$e_L = c$$

- **The returns to scale are measured by the sum of the exponents.**

If  $b + c > 1$  then there are increasing returns.

If  $b + c < 1$  then there are decreasing returns.

If  $b + c = 1$  then there are constant returns.

If

$$X_0 = aK^b L^c$$

and

$$X^* = a(kK)^b (kL)^c$$

then

$$X^* = k^b k^c aK^b L^c$$

$$X^* = k^{b+c} X_0$$

Where k is some constant.

- **the elasticity of substitution for Cobb-Douglas functions = 1**

$$\sigma = \frac{\frac{\partial(K/L)}{K/L}}{\frac{\partial(\partial K / \partial L)}{\partial K / \partial L}}$$

where  $\sigma$  = elasticity of substitution.

So

$$\partial K / \partial L = MRTS_{L,K} = -\frac{c}{b} \cdot \frac{K}{L}$$

so

$$(\partial K / \partial L) / (K / L) = -\frac{c}{b}$$

and

$$\partial(\partial K / \partial L) / \partial(K / L) = -\frac{c}{b}$$

thus

$$\sigma = 1$$

This holds irrespective of return to scale characteristics.

You might also note that if the first order conditions for cost

minimisation are satisfied then

$$(\partial K / \partial L) = -(w / r)$$

and

$$\partial(\partial K / \partial L) = -\partial(w / r)$$

so

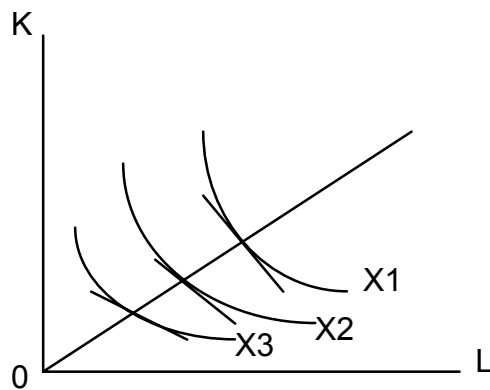
$$\sigma = \frac{\frac{\partial(K/L)}{K/L}}{\frac{\partial(w/r)}{(w/r)}} = 1$$

So, a 10% change in a factor price leads to a 10% change in the factor input ratio in the opposite direction.

eg. a 10% increase in w relative to r leads to a 10% reduction in L relative to K.

### c) **Technological Progress**

Capital deepening technical progress occurs, if long a line on which the KL ratio is constant, the  $MRTS_{L,K}$  increases. This implies that the MP of K increases by more than the MP of L. The ratio of the MPs (which is the MRTS) decreases in absolute value, but as the slope of the isoquant is negative, this sort of technical progress increases the  $MRTS_{L,K}$ . The slope of the shifting isoquant becomes less steep along any given radius.

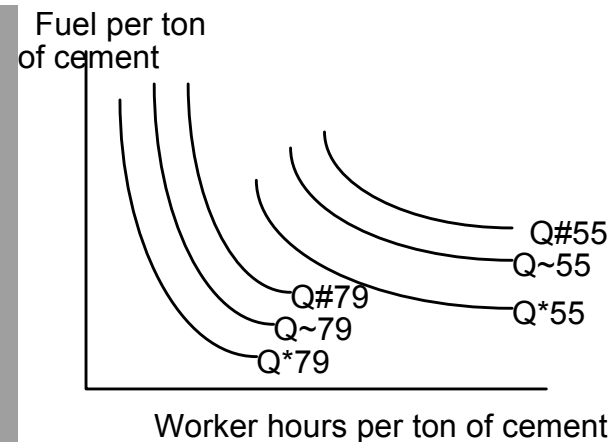


Can you draw isoquants depicting labour deepening and neutral technical progress?

**Technology shifts the isoquants**

Forsund FR & Hjarlmarsson L (1983) Technical progress and structural change in the Swedish cement industry, 1955 - 1979, *Econometrica* 51, no. 5 pp. 1449-67.

As part of their study these authors showed the short run substitution possibilities between labour and fuel for each year from 1955 to 1979 for one large Swedish cement company.



The above diagram shows isoquants for 3 different levels of capacity. Strictly, each set belongs on its own graph. However, the diagram illustrates 2 important points.

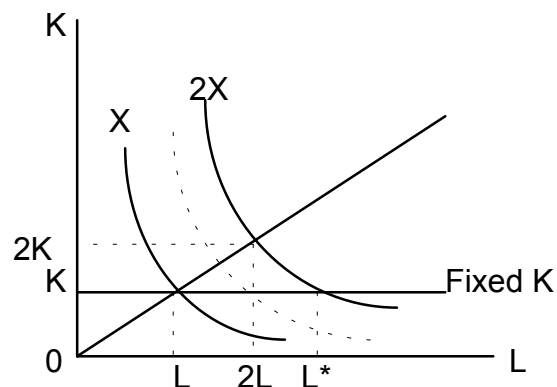
As new kilns were employed there was an overall increase in productivity. The average amount of labour to produce the same amount of cement decreased by 67% over the period under study. The amount of energy required dropped by 17%.

The slope of the isoquants changed. The MRTS became steeper and there was a decrease in the possibility for substitution between fuel and labour. Thus, the new kiln favoured a higher use of energy than labour at each output level.

**Returns to Scale and the Law of Diminishing Returns**

The generality of the Law of Diminishing Returns is such that it should overcome return to scale effects and this is usually

the case.



In the above case of constant returns to scale, we can see that doubling inputs should lead to double the output but with a fixed  $K$  output falls as we double  $L$ . Thus, diminishing returns to the variable factor prevail.

Can you describe what happens in the decreasing and increasing returns cases?

### **Reading**

Nicholson W (1997) *Intermediate Microeconomics and its Application*, Harcourt Brace and Co. Orlando.

Schotter A (1997) *Microeconomics. A Modern Approach*, Harper Collins, New York.

Mulligan (1992) *Managerial Economics*  
Koutsoyiannis A (1979) *Modern Microeconomics*, Macmillan, London

## Questions: Some Basics on Production Functions

### Questions for the Lecture

1. If the production function is of the type

$Q = f(K, L)$ , where  $Q$  is output,  $K$  is capital and  $L$  is labour and  $K$  is fixed in the short run define the law of diminishing returns.

2. Graph the short run total product curves for each of the following short run production functions if  $K$  is fixed at  $K_0 = 4$

$$Q = 2K + 4L$$

$$Q = K^2 L^2$$

$$Q = KL$$

$$Q = K L^{1/2}$$

Do these functions obey the law of diminishing returns?

3. Frisbees are produced according to the production function

$$Q = 2K + L$$

Where  $Q$  = output of Frisbees per hour  
 $K$  = capital input per hour

$L$  = labour input per hour

- (i) if  $K = 10$ , how much  $L$  is needed to produce 1000 frisbees per hour?
- (ii) If  $k = 25$  how much  $L$  is needed to produce 100 frisbees per hour?
- (iii) Graph the  $Q = 100$  isoquant. Indicate the points on that isoquant defined in (i) and (ii). What is the Marginal rate of substitution along this isoquant? Explain why  $MRTS_{L,K}$  is the same at every point along the isoquant?
- (iv) Graph the isoquant of  $Q = 50$ . What can you say about returns to scale in this firm?
- (v) Suppose technical progress resulted in the production function for frisbees becoming

$$Q = 3K + 1.5L$$

Answer parts (i) through to (iv) again and discuss how it compares with the previous case.

4. Consider the following production functions

$$Q = (K + L)^{1/2}$$

$$Q = (K + L)^2$$

$$Q = K + L$$

Graph the isoquants for each using  $Q = 5$  and  $Q = 10$  for the first and third functions and  $Q = 25$  and  $Q = 100$  for the

second function. What can you say about the returns to scale associated with each?

5. Capital and labour are used in fixed proportions to produce airline flights – it takes two operators (pilots) and one plane for each trip made. Technical and safety problems make it impossible for a single pilot to fly a plane.

- (i) What is the output of this production process and what do the isoquants look like?
- (ii) Suppose an airline hired 30 pilots and 10 planes during a particular period. Explain both graphically and in words why it might be a foolish thing to do.
- (iii) Suppose progress in avionics equipment made it possible for a single pilot to handle each plane. How would that shift the isoquant map described in part (i)? Would this raise the average productivity of labour in this industry? Would it raise the average productivity of capital (planes)? Explain.

6. Suppose that the hourly output of mustard ( $Q$ ) is characterised by

$$Q = 20\sqrt{KL}$$

where  $K$  is the number of large pots used each hour and  $L$  is the number of worker hours employed.

- (i) Graph the  $Q = 2000$  kilos per hour isoquant.

(ii) The point  $K = 100, L = 100$  is one point on the  $Q = 2000$  isoquant. What value of  $K$  corresponds to  $L = 101$  on that isoquant? What is the approximate value for the  $MRTS_{L,K}$  at  $K = 100, L = 100$ ?

(iii) The point  $K = 25, L = 400$  also lies on the  $Q = 2000$  isoquant. What value of  $K$  corresponds to  $L = 401$  on that isoquant? What is the approximate value for the  $MRTS_{L,K}$  at  $K = 25, L = 400$ ?

(iv) For this production function, it can be shown that the general formula for the  $MRTS_{L,K}$  is

$$MRTS_{L,K} = K/L$$

Compare the results from applying this formula to those calculated in (ii) and (iii). To convince yourself further, perform a similar calculation for the point  $K = 200, L = 50$ .

(v) If technical progress shifted the production function to

$$Q = 40\sqrt{KL}$$

All of the input combinations identified earlier can now produce  $Q = 4000$  pounds per hour. Would the various values calculated for the  $MRTS_{L,K}$  be changed as a result of this technical progress, assuming now that the  $MRTS_{L,K}$  is to be measured along the  $Q = 4000$  isoquant?

7. Pearl and Enos (1975) estimated the following production function

$$Y = 24.95H^{0.36} D^{1.72}$$

where

Y= flow of oil in barrels per day

H= hydraulic horsepower of a pump

D= internal pipe diameter in inches

- (i) Show the isoquants for

$$Y = 50000$$

$$Y = 100000$$

$$Y = 150000$$

barrels per day and comment on their shape.

- (ii) Calculate the marginal products of H and D and comment on their meaning.
- (iii) Calculate the returns to scale for this production function. Are they increasing, decreasing or constant?

## Seminar Questions

1. Plot TP, MP and AP curves for labour for one unit of land from the table below and explain the 3 stages of production.

Land	1	1	1	1	1	1	1	1	1
Labour	0	1	2	3	4	5	6	7	8
Total Product of labour for one unit of land	0	3	8	12	15	17	17	16	13

2. What is meant by the phrase constant returns to scale?
3. We can show that under constant returns to scale there is perfect symmetry between the stages of production for land and labour.

Look at the Table in question 1. We can see that 8 units of labour on 1 unit of land produces 13 units of output; therefore, using 1/8 of the quantity of labour and land should result in 1/8 of 13 units of output, under constant returns to scale. Thus, if only one unit of labour was used and land could be varied, 1/8 of a unit of land would give rise to 13/8 units of output. The Table below shows the outcomes of this process.

Land	1	1/2	1/3	1/4	1/5	1/6	1/7	1/8	
Labour	1	1	1	1	1	1	1	1	
Total Product of land for one unit of labour	3	4	4	15/4	17/5	17/6	16/7	13/8	

(i) Calculate the AP and MP for land given one unit of labour.

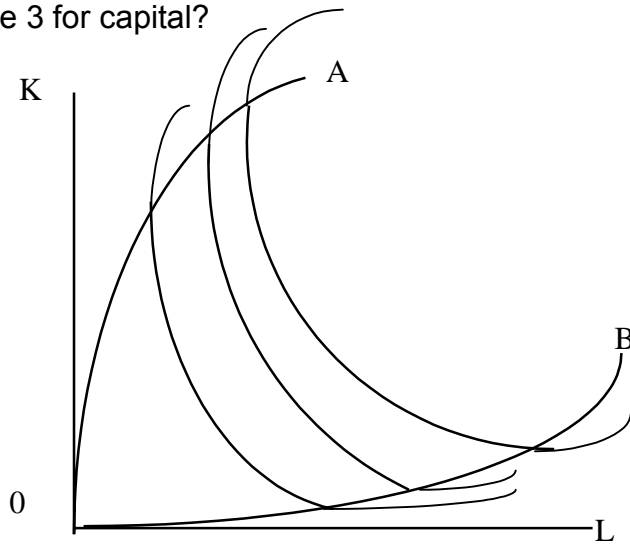
(ii) Presumably your graph for question 1 showed an increasing labour/land ratio on the horizontal axis. I want you to superimpose the TP, AP, and MP curves of land/labour ratio on this graph.

(iii) What can you say about the stages of production for labour and land from this 'new' graph?

4. With reference to stage 2 of production, why does the producer operate in stage 2? What factor combination (within stage 2) will the producer actually use? Where will the producer operate if the price of labour is zero? If the price of land is zero? If the price of land equals the price of labour?
5. If the amount of a fixed factor increased what would happen to the TP, AP and MP curves?

6. Now assume we have 2 variable inputs, labour and capital. What is meant by the marginal rate of technical substitution of labour for capital?

7. The diagram below shows ridge lines, which is where the  $MRTS = 0$ . Why to the right of ridge line 0B do we have stage 3 for labour? Why above ridge line 0A do we have stage 3 for capital?



8. Assuming constant returns to scale, define stages 1,2 and 3 for labour and capital in the above diagram.

(i) Explain why a movement down an isoquant (within the ridge lines) implies that the MP of labour is declining?