

UNIT EC425, LEVEL 2

COSTS

**MICROECONOMICS 2:
ECONOMICS AND ORGANISATIONS**

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Costs

How do Economists define cost?

Opportunity Cost

The foregone value associated with current rather than next-best use of an asset

Economists incorporate normal profit into their definition of cost because this reflects the owners willingness to stay in that activity rather than go into their next form of employment. So, when a firm declares 'large' profits these may not reflect monopoly power, they may reflect a 'normal return to the owners; that is, they are a cost to the business.

Of course, it is entirely different if a firm has obtained large profits because they have exploited their market position. Then economists distinguish between 'normal' and 'abnormal' profits, the latter being a cost to customers.

Of course, other cost concepts are relevant to economists, in particular,

- incremental costs and
- sunk costs

However, these will be explored at another time.

Why are costs relevant?

Some examples include:

- price-output decisions
- barriers to entry
- market structure
- growth policy of the firm (mergers, vertical integration, etc.)

The above have relevance in the debate between productive and allocative efficiency and thus have a direct link to whether organisations should be regulated.

For the moment though we shall concern ourselves with productive efficiency issues. Specifically, I will

- examine the traditional link between production and costs
- consider the importance of economies of scale and scope.

The Link between Production and Costs

As we are measuring inputs as flows then we should note that the costs of K and L are

- r - the rental price of K per time period (the rental price of K is the given by what the organisation forgoes, per hour, or per week, by owning the machine. It is not the price of a new piece of K)
- w - wages per time period

Total Cost, C , is given as

$$C = rK + wL$$

The cost function, however, describes how the minimum total cost of production, C , changes with the volume of output and input prices.

$$C = F(X, r, w)$$

We assume, for the purposes of simplicity that factor prices are determined in perfectly competitive markets and are given. We are also assuming a long run situation.

Equilibrium for the Organisation

A firm can either maximise profit subject to an output constraint or it can minimise costs subject to the constraint imposed by the production function.

Formally,

$$\text{Minimise } C = rK + wL$$

$$\text{s.t. } X = f(K, L)$$

forming the composite function

$$\phi = rK + wL - \lambda[\bar{X} - f(K, L)]$$

Assuming the second order conditions for a minimum are satisfied the necessary conditions are as follows

$$\frac{\partial \phi}{\partial K} = r - \lambda \frac{\partial f(K, L)}{\partial K} = 0 = r - \lambda \frac{\partial X}{\partial K}$$

$$\frac{\partial \phi}{\partial L} = w - \lambda \frac{\partial f(K, L)}{\partial L} = 0 = w - \lambda \frac{\partial X}{\partial L}$$

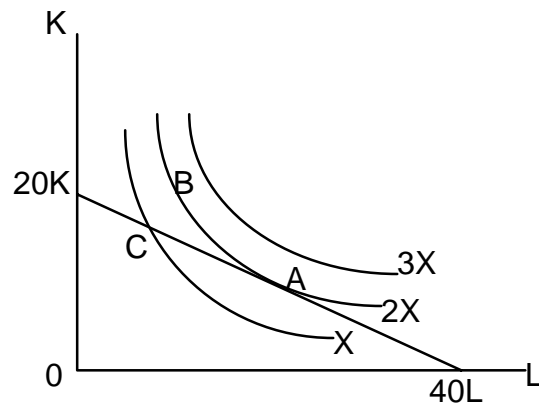
$$\frac{\partial \phi}{\partial \lambda} = -[\bar{X} - f(k, L)] = 0$$

Dividing through

$$\frac{w}{r} = \frac{\partial X / \partial L}{\partial X / \partial K} = MRTS_{L,K}$$

That is costs are minimised where the MRTS between inputs in production is equal to the ratio of factor prices. Or where the slope of the isoquant equals the slope of the isocost curve.

An intuitive grasp of the equilibrium concept can be gained via the following example. If an organisation has an outlay of £2,000 and the prices of L and K are £50 and £100 respectively. Then we can draw an isocost curve like that set out below



Note that if

$$C = rK + wL$$

Re-arrangement gives

$$K = \frac{C}{r} - \frac{w}{r}L$$

which, for a given outlay, C_0 , means a straight line with intercept C_0/r and slope $-w/r$ (i.e. $dK/dL = -w/r$).

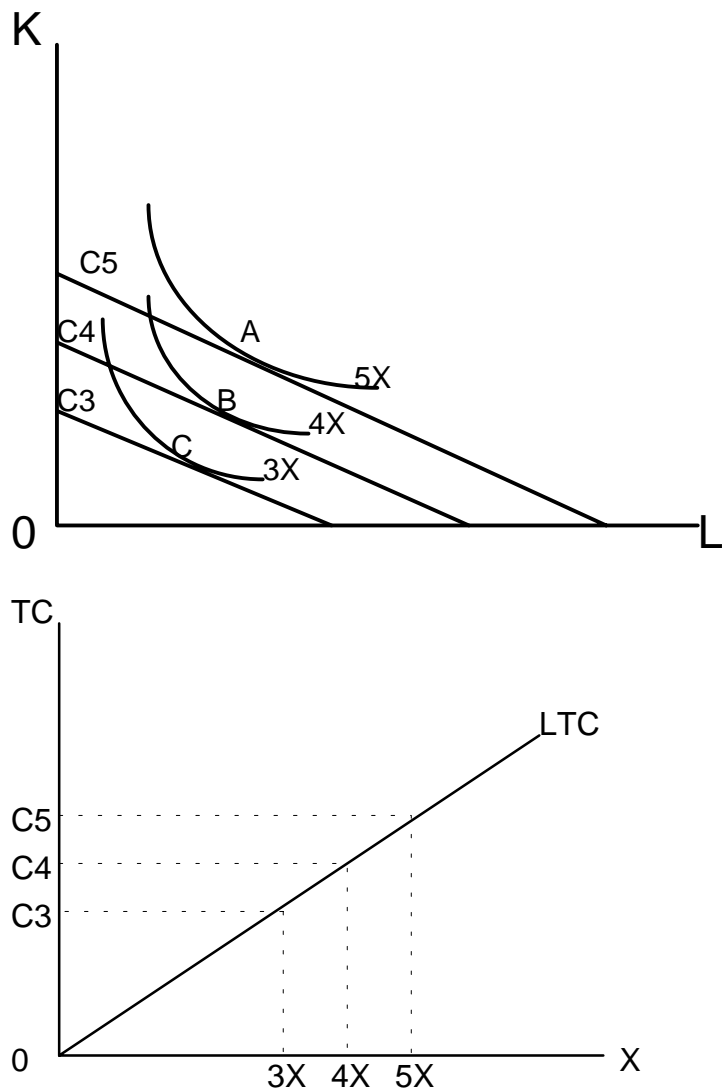
For the diagram above, the intercept is 20 and the slope -0.5.

Equilibrium is at point A.

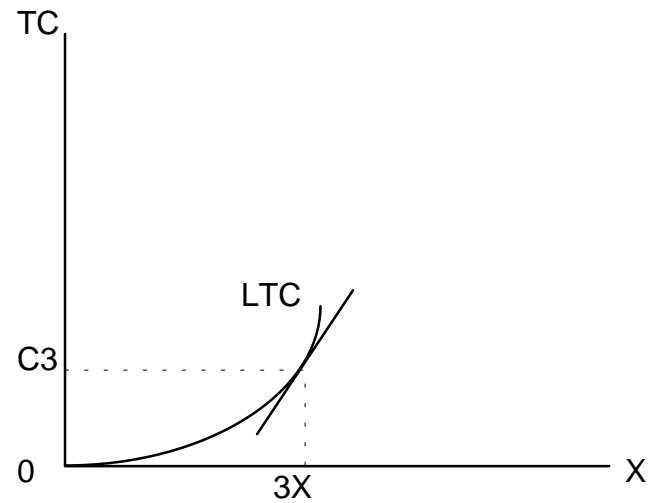
You should confirm for yourselves why B and C are not equilibrium points.

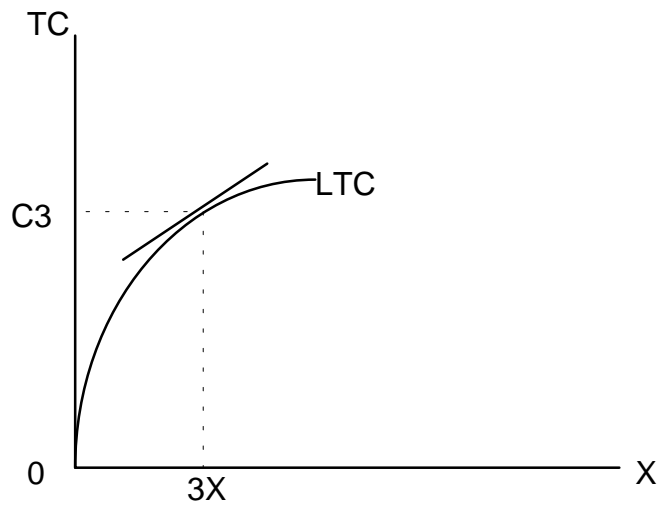
Expansion Path

The optimal expansion path for the firm depends on the ratio of factor prices. If the production function is homogeneous then the expansion path will be a straight line. In the diagram below, the expansion path involves constant returns to scale and so the resulting total cost curve is a straight line.

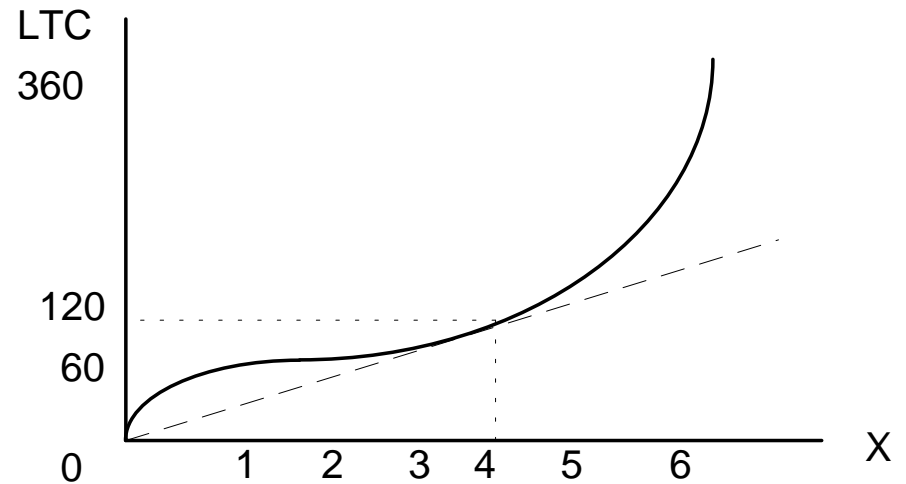
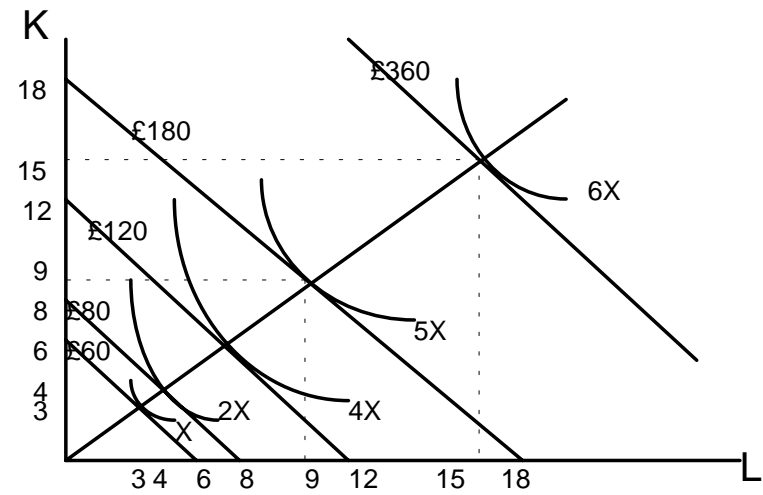


Clearly, if there are decreasing returns to scale then the total cost curve rises at an increasing rate, whilst increasing returns to scale indicate costs rising but at a diminishing rate. The next 2 diagrams show this.





In the figures below the link between isoquants with variable returns to scale and the long run cost curves are shown.



Cobb-Douglas

As the Cobb-Douglas production function is homogeneous, its expansion path will be a straight line.

We know that for the CD

$$\frac{c}{b} \cdot \frac{K}{L} = MRTS_{L,K}$$

and

$$MRTS_{L,K} = -\frac{w}{r}$$

so

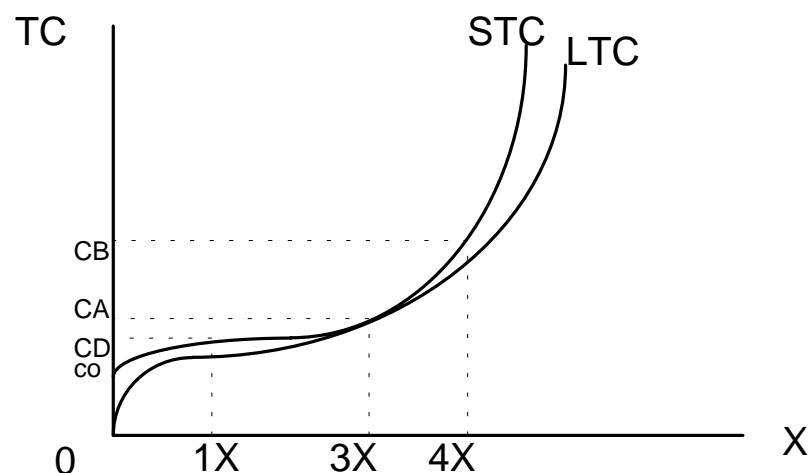
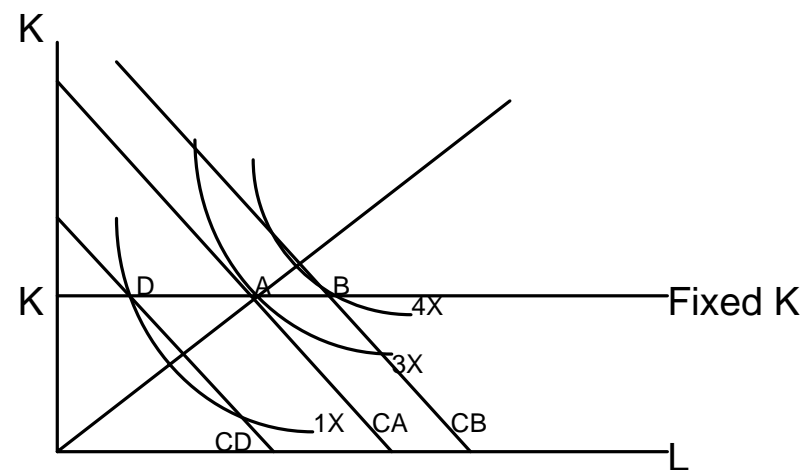
$$\frac{K}{L} = -\frac{bw}{cr}$$

Which for given input prices implies that KL ratio is the same at every tangency between isoquants and isocost lines.

Short Run Cost Function

$$C = f(X, P_f, \bar{K})$$

where one factor, usually K, is fixed.



Some Definitions

$$AFC = \frac{TFC}{X}$$

$$AVC = \frac{TVC}{X}$$

$$ATC = \frac{ATC}{X} = AFC + AVC$$

$$MC = \frac{\partial TC}{\partial X} = \frac{\partial TVC}{\partial X}$$

Why is the AVC curve U-shaped?

With L as the only variable input

$$AVC = \frac{TVC}{X} = \frac{wL}{X} = \frac{w}{X/L} = \frac{w}{AP_L}$$

Since AP_L usually rises, reaches a maximum, then falls, with w constant, it follows that AVC first fall, reaches a minimum, then rises

Why is MC curve u-shaped?

$$MC = \frac{\partial TC}{\partial X} = \frac{\partial wL}{\partial X} = \frac{w}{\partial X / \partial L} = \frac{w}{MP_L}$$

Reasoning similar to that for AVC.

Note rising AVC and MC reflects diminishing average and marginal returns respectively.

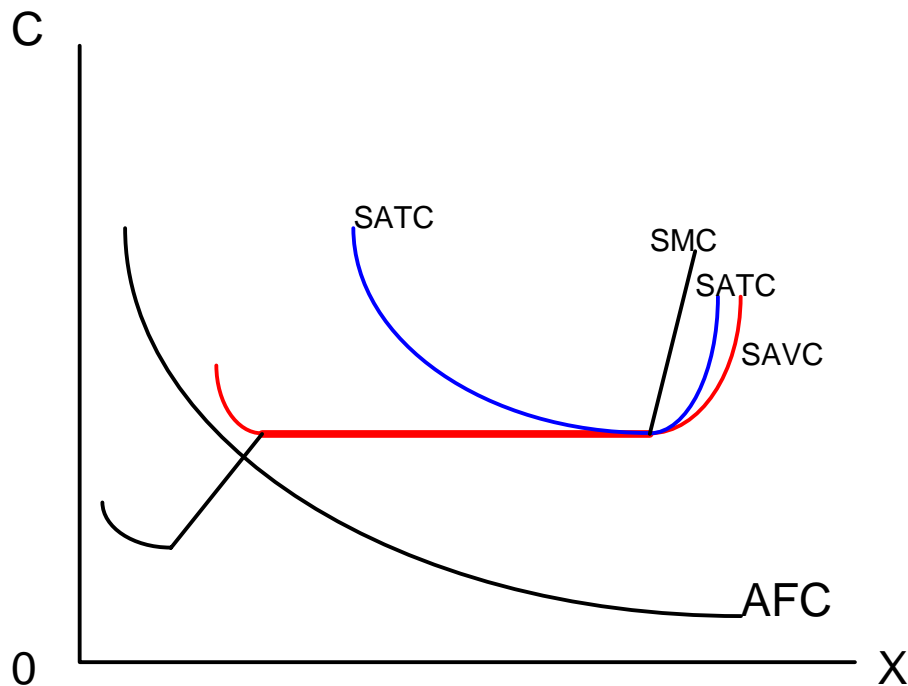
The Current View of Short Run Cost Curves

A flat AVC reflects 'reserve capacity'.

Firms plan for flexibility in recognition of seasonal, cyclical and other influences.

Firms bring in more fixed capital when it hires labour so that KL ratio and even MC remain constant in the short run.

Evidence: Dean (1950) Johnson (1960)



Long Run Cost Function

$$C = f(X, T, P_f)$$

Cost Elasticity?

$$\epsilon_c = \frac{\partial TC}{\partial X} \times \frac{X}{TC} = MC \times \frac{1}{AC}$$

If

- $\epsilon_c > 1$ Then increasing returns prevail
- $\epsilon_c < 1$ Then decreasing returns prevail
- $\epsilon_c = 1$ Then constant returns prevail

Economies of Scale

Economies of scale are distinguished into real and pecuniary economies.

- Pecuniary economies are realised from paying lower factor prices, and strictly speaking do not imply a reduction in the quantity of inputs used.

Whilst paying lower prices for raw materials via bulk buying or obtaining lower interest rates are important sources of size advantage, paying lower wages is rare unless the firm has monopsonistic advantages.

- Real economies are associated with a reduction in the physical quantity of inputs as the firm expands.

There are 4 types of real economies

- production economies
- selling and marketing economies

- managerial economies
- transport and storage economies

The latter become particularly important when discussing inter-plant economies of scale, rather than intra-plant scale economies. Thus, the issue of multi-plantness is an important element of understanding efficient production. Multi-Plantness arises where markets are dispersed and high transport costs exist because weight - value problems. Transport costs can have a large impact on the optimal scale of the single plant and so may give rise to multi-plantness.

Another reason for multi-plantness is that of Pooling risks. If risks are not systematically related then there may be opportunities to hold less slack capacity or inventories to guard against contingencies than a single plant would. This may lower costs of raising capital.

However there may be diseconomies from operating multiple plants because of co-ordination and communication breakdown.

A particularly important real (production) economy is that of learning effects.

Learning Effects

The efficiency of a plant increases over time with learning. More formally,

$$L_i = kQ^{-\gamma}$$

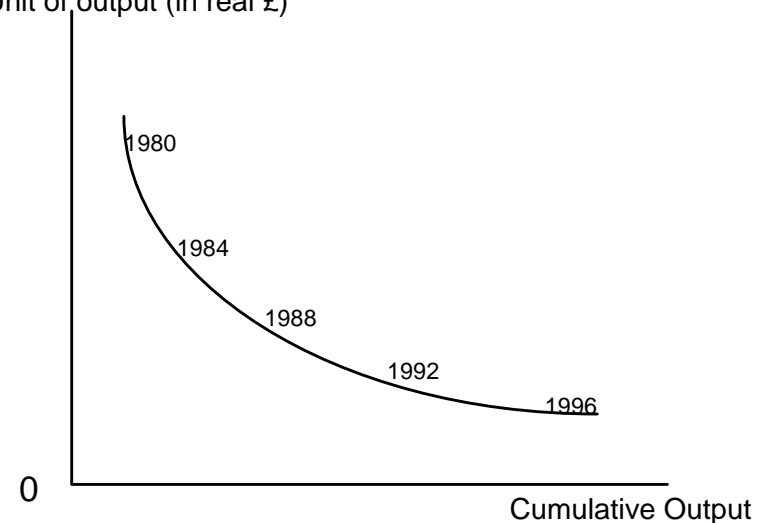
L_i = Labour input to produce i

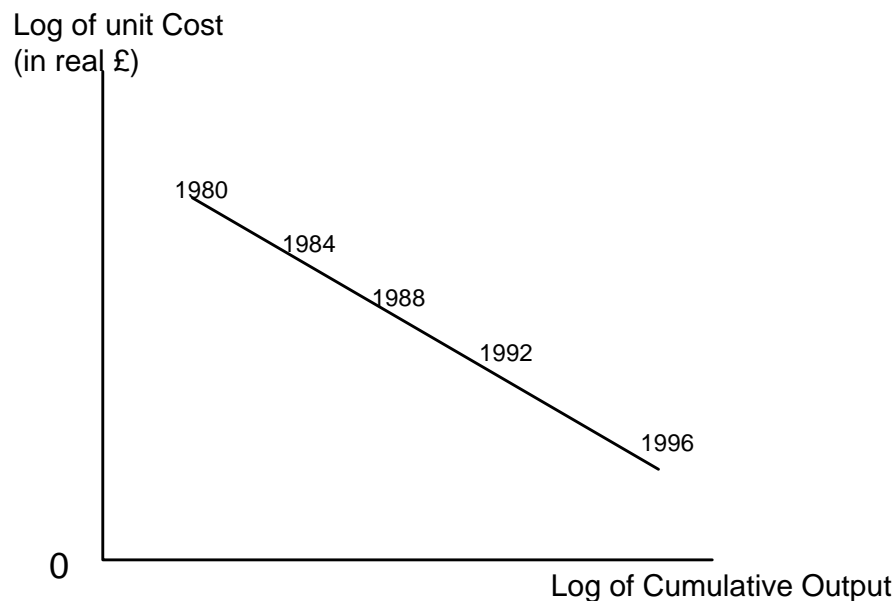
Q = Cumulative output

k = constant

γ = learning effect

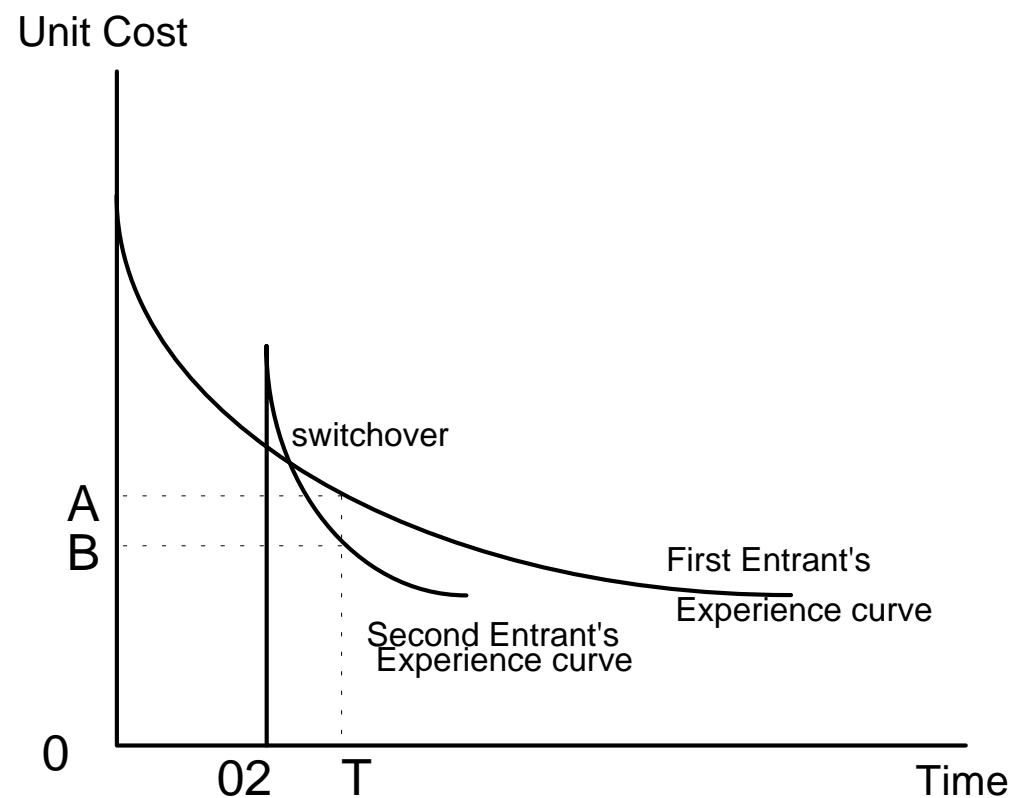
National Cost per
Unit of output (in real £)





Boston Consulting Group showed the importance of learning curves in various industries in the 1970s. However, they did not divorce learning from scale effects.

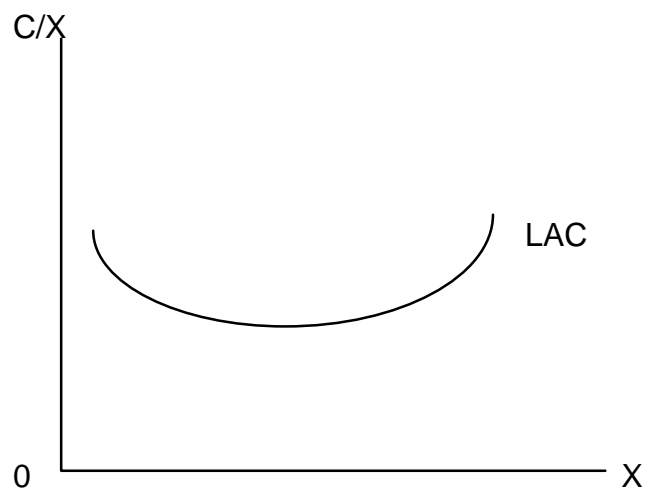
These learning effects are an important source of intra-industry competitive advantage as the diagram below shows. (Note that shallower learning curves may reflect mature products.)



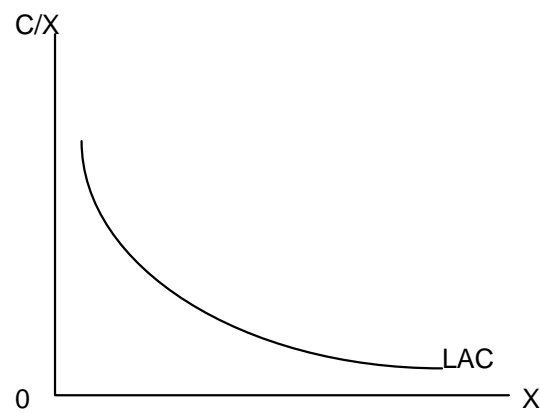
Economies of scale give rise to downward sloping LAC curves. The following depict the possible shapes of the LAC. In the first, the possibility of diseconomies arising from the breakdown of managerial control, co-ordination and communication leads to a rising LAC after a certain level of output.

In the second diagram the existence of scale economies over all levels of output is often referred to as a natural monopoly (though note my lecture on this subject later). Incidentally, the possibility of a protected monopoly may give rise to **X-inefficiency**, as identified by Liebenstein, which indicates the waste of internal scarce resources and thus higher than necessary average costs.

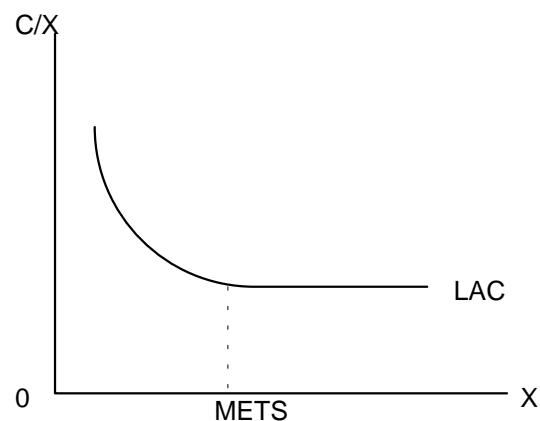
Traditional view of the Long Run Average Cost Curve



A Declining Long Run Average Cost Curve: Sufficient Conditions for a Natural Monopoly



A modern view of the Long Run Average Cost Curve



The Minimum Efficient Technical Size of a firm is an important concept in industrial economics. It shows, under the assumption that firms have access to similar technologies, the minimum output level that a firm has to achieve before it becomes competitive.

Evidence on (real) scale economies comes from numerous sources. Pratten (1987) has provided some estimates for the UK via interviews and analysis of the technology used in specific sectors.

We can use these estimates to obtain the optimal number of firms for an industry. But note that the size of the market under consideration is important.

Some Products for which the Minimum Efficient Technical Size is superior equal to 20% of the production of the UK

<u>NACE Code</u>	<u>Product</u>	<u>METS as % of Production</u>		<u>Cost gradient at ½ METS</u>
		<u>UK</u>	<u>EC</u>	
351	Cars	200	20	6 - 9
33	Computers	>100	n.a.	5
364	Aircraft	>100	n.a.	20
321	Tractors	98	19	6
251	Titanium Dioxide	63	50	8 - 16
346	Washing Machines	57	10	4.5
345	TV Sets	40	23	5
429	Cigarettes	24	6	1.4
251	Synthetic Rubber	24	3.5	15
251	Petrochemicals	23+	3+	12
326	Ball Bearings	20	2	8 - 6

Source: Adapted from Emerson *et al* 1988, *The Economics of 1992*, Oxford University Press. The original source is cited as Pratten (1987).

Scope Economies

Scale economies refer to single-product firms, but most organisations offer multiple products or services. Thus, economists discuss economies of scope, or joint production.

Why joint production? In the 2 product case

$$C(x_1, x_2) < C(x_1, 0) + C(0, x_2)$$

All the above means is that it is cheaper for one organisation to produce x_1 and x_2 , than it is for 2 independent firms to provide these same two good (or services) independently.

3 kinds of scope economy

1. Factors of production are 'public'. i.e. once acquired they can be used in production of other goods. e.g. peak and off-peak electricity.
2. Where spare capacity exists firms need to find alternative outlets for expensive inputs. (Spare capacity might exist because of large capital 'indivisibilities' so that efficient output is greater than market size. Additionally, imperfect competition may lead to profit maximising behaviour at a level below full capacity.)
3. Cost complementarities, i.e. if the MC of one product falls as the output of another increases. e.g. chemicals.

Reading

Nicholson W (1997) Intermediate Microeconomics and its Application, Dryden Press, seventh edition, Orlando.

Ferguson C E (1972) Microeconomics, Third Edition, Irwin, Homewood, Illinois.

Problems

1. A pottery firm has an infinitely substitutable production function of the form

$$q = 2K + L$$

- (i) Graph the isoquant maps for $q = 20$, $q = 40$, $q = 60$. What is the $MRTS_{L,K}$ along these isoquants?
- (ii) If the wage rate, w , is £1 and the rental rate on capital, r , what is the cost minimising combination of K and L will the manufacturer employ for the different production levels in (i)? What is the manager's expansion path?
- (iii) Would your answer to (ii) change if $r = £3$ and $w = £1$?

2. The long run total cost curve of a firm is given by

$$TC = q^3 - 40q^2 + 430q$$

Where q is output per week.

- (i) What is the general shape of this total cost function?
- (ii) Calculate the average cost function for this firm. What shape does the graph of this function have? At what output does average cost reach a minimum? What is the average cost at this level of output?

- (iii) Calculate the marginal cost for this firm. Show that marginal cost intersects the average cost at its minimum point.
- (iv) Graph marginal and average costs.

3. Seashells are collected on a tropical island according to the following production function

$$q = 2 \sqrt{H}$$

Where q is the number of shells collected per day and H is the number of hours spent by employees finding these shells. The employees are paid £8 per hour.

- (i) Calculate the total and average cost curves (as a function of q).
- (ii) What is the total cost per day if 40 shells are collected? 60 shells? 80 shells? What is the average cost for these same outputs?

4. A firm producing hockey sticks has a production function given by

$$Q = 2 \sqrt{KL}$$

In the short run the firm's capital is fixed at $K = 100$. The rental rate, r , is £1, and the wage rate, w , is £4.

- (i) Calculate the firm's short run total cost curve. Calculate the short run average cost curve.
- (ii) What is the firm's short run marginal cost curve? If $q = 25$, what are the short run total costs, the short run average cost and the short run marginal cost? Perform the same calculations for $q = 50$, $q = 100$, $q = 200$.
- (iii) Graph the SRAC and SRMC curves for the firm. Indicate the points found in (ii).
- (iv) Where does the SRMC intersect the SRAC curve? Explain.

5. For the following functions show the associated levels of output on an isoquant map and graph the shape of the total cost function.

- (i) $q = 2K^{1/2} L^{1/2}$
- (ii) $q = 2K^{1/2} L$
- (iii) $q = 2K^{1/2} L^{1/4}$

Assume that $w = r = £1$.

6. If $Q = 10K^{0.5} L^{0.5}$
And $r =$ rental price of capital and $w =$ price of labour
- (i) Write an expression for the total cost function, the average cost function and the marginal cost function.
 - (ii) If $w = r = £4$ what is total cost, average cost and marginal cost?
 - (iii) If $w = 4$ and $r = £9$ what is TC, AC and MC? Explain what is happening.

7. Assume that a firm produces 90 units of output using 9 units of input X and 9 units of input Y. The firm's technological possibilities can be represented by the function

$Q = 10K^{1/2} L^{1/2}$ whose marginal products are $MP_x = Q/2X$ and $MP_y = Q/2Y$.

- (i) If the price of X is £8 and the price of Y is £16, is the input combination of 9 units of X and 9 units of Y the most efficient way to produce 90 units of output?
- (ii) What must be the ratio of input prices for this input combination to be efficient?
- (iii) Assume that the price of X is £1 and the price of Y is £2. Derive the least cost way to produce 400 units of output.

8. A hospital produces health services using two inputs: hospital beds and labour. There is a government regulation restricting the number of beds to B. Assume that the hospital is currently using B beds and L units of labour services to produce Q_1 units of health services. Also assume that the hospital plans to expand its output to Q_2 units of health services. Prepare a diagram to show how this government regulation restricting the number of beds would affect the efficiency of delivering health services.

9. Suppose that a firm has long run total costs of £1000 for producing 100 units of output. The two inputs for production are labour and capital. Labour costs £10 per unit

and capital costs £10 per unit. The firm is currently producing 100 units of output and is using the cost minimising combination of 50L and 50K for labour and capital.

- (i) On an isoquant diagram, show that an increase in output from 100 units to 150 units will result in higher short run than long run total costs, average costs and marginal costs.
- (ii) Show that a decrease in output from 100 units to 50 units will result in higher short run than long run total costs and average costs but higher long run than short run marginal costs.
- (iii) Give an intuitive explanation for these relationships between short run and long run cost curves.

10. Give an intuitive explanation of a fall in the price of labour relative to that of capital.